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A multi-item mixture inventory model involving random lead time and demand with budget constraint and surprise function

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ABSTRACT

This study deals with a multi-item mixture inventory model in which both demand and lead time are random. A budget constraint is also added to this model. The optimization problem with budget constraint is then transformed into a multi-objective optimization problem with the help of fuzzy chance-constrained programming technique and surprise function. In our studies, we relax the assumption about the demand, lead time and demand during lead time that follows a known distribution and then apply the minimax distribution free procedure to solve the problem. We develop an algorithm procedure to find the optimal order quantity and optimal value of the safety factor. Finally, the model is illustrated by a numerical example.

1. Introduction

Since the formulation of the classical economic order quantity (EOQ) by Harris [1], lot of research papers, news letters and reviews dealing with various aspects of inventory control and production systems with single and multi-objectives have been published in different journals throughout the world. In most of the work, the author optimizes the objective values which consist of setup cost, holding cost, Stockout cost, etc. Stockout cost is usually the most difficult inventory cost to ascertain. Stockout cost may be due to backorders or lost sales, and it may be expressed on a per unit basis, a per outage basis, or some other basis.

Lead time management is an important issue in production and operations management. According to Tersine [2], the following four different situations may arise:

(1) constant demand and constant lead time,
(2) variable demand and constant lead time,
(3) constant demand and variable lead time and
(4) variable demand and variable lead time.

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In the fourth case where both demand and lead time uncertainties are accounted for simultaneously, a joint probability distribution can be created that gives the probabilities for various combinations of demand level and lead time length. In this area, Ord and Bagchi [3], Burgin [4] consider Normal demand and Gamma lead time; Das [5] takes model Normal demand and exponential lead time and Carlson [6] assumes that demand is Poisson and lead time is Exponential. Recently, following Liao and Shyu [7] many authors such as Ben-Daya and Raouf [8], Ouyang et al. [9], Ouyang and Chuang [10], Chu et al. [11] Park [12], Eynan and Kropp [13] considered lead time as a variable and controlled it by paying extra crashing cost. Moreover, Gallego and Moon [14] give the idea for determining the expected amount of stockout quantity when the demand distribution is unknown but the mean and variance are known.

But till now, none has considered multi-item inventory problem with both the demand and lead time random, where distribution of demand and lead time is unknown. Moreover, to handle fuzzy chance-constrained programming technique, the surprise function approach is used in this model. This environment is considered in our present model.

Thus, this article deals with a multi-item stochastic inventory model in which safety factor and order quantity are considered as the decision variables. To find the expected amount of stockout quantity we relax the assumption about the distribution of the demand during lead time. A random budget constraint is also added to this model. After that, we transform the optimization problem with random budget constraint into a multi-objective optimization problem with the help of fuzzy chance-constrained programming technique and surprise function. In our studies, the multi-objective optimization problem is then solved by using the minimax distribution free procedure. We then proposed an iterative procedure to find the optimal order quantities and optimal safety factors. A numerical example is presented to illustrate the result of the proposed model.

2. Random sum and its application

Considering N mutually independent identically distributed random variables $\widehat{X}_1, \widehat{X}_2, \widehat{X}_3, \ldots, \widehat{X}_N$ with distribution functions $F(x)$, mean $E[\widehat{X}]$ and variance $Var[\widehat{X}]$, where N is a fixed constant and then

$$\widehat{T} = \widehat{X}_1 + \widehat{X}_2 + \widehat{X}_3 + \cdots + \widehat{X}_N. \tag{1}$$

If N be itself a random variable then the probability mass function of the discrete random variable, $p_{\widehat{N}}(n)$ is assumed to be given. For a fixed value of $\widehat{N} = n$, the conditional expectation of $\widehat{T}$ is easily obtained

$$E[\widehat{T}/\widehat{N} = n] = \sum_{i=1}^{n} E[\widehat{X}_i] = nE[\widehat{X}]. \tag{2}$$

Then, using the theorem of total expectation the following result is obtained

$$E[\widehat{T}] = \sum_n nE[\widehat{X}]p_{\widehat{N}}(n) = E[\widehat{X}]\sum_n np_{\widehat{N}}(n) = E[\widehat{X}]E[\widehat{N}]. \tag{3}$$

In order to obtain the $Var[\widehat{T}]$, I first compute $E[\widehat{T}^2]$, which is as follows:

$$E[\widehat{T}^2/\widehat{N} = n] = Var[\widehat{T}/\widehat{N} = n] + \left\{E[\widehat{T}/\widehat{N} = n]\right\}^2 \tag{4}$$

$$\text{but} \quad Var[\widehat{T}/\widehat{N} = n] = \sum_{i=1}^{n} Var[\widehat{X}_i] = nVar[\widehat{X}]. \tag{5}$$

Substituting (2) and (5) in (4) the following result is derived

$$E[\widehat{T}^2/\widehat{N} = n] = nVar[\widehat{X}] + n^2\left\{E[\widehat{X}]\right\}^2. \tag{6}$$

Now using the theorem of total moments it is derived that

$$E[\widehat{T}^2] = \sum_n \left\{nVar[\widehat{X}] + n^2\left(E[\widehat{X}]\right)^2\right\}p_{\widehat{N}}(n) = Var[\widehat{X}]E[\widehat{N}] + E[\widehat{N}^2]\left(E[\widehat{X}]\right)^2. \tag{7}$$

Finally

$$Var[\widehat{T}] = E[\widehat{T}^2] - \left(E[\widehat{X}]\right)^2 = Var[\widehat{X}]E[\widehat{N}] + E[\widehat{N}^2]\left(E[\widehat{X}]\right)^2 - \left(E[\widehat{X}]\right)^2\left(E[\widehat{N}]\right)^2 = Var[\widehat{X}]E[\widehat{N}] + \left(E[\widehat{X}]\right)^2 Var[\widehat{N}]. \tag{8}$$

Application: Let daily demand of any item in a showroom be random and denoted by $\widehat{D}_i$ (demand of the i-th day) with mean $E[\widehat{D}]$ and Variance $Var[\widehat{D}]$; after r day from the beginning of the cycle an order of that item is placed; and let L be the length of the lead time. Then the DDLT is

$$\widehat{X} = \widehat{D}_{r+1} + \widehat{D}_{r+2} + \widehat{D}_{r+3} + \cdots + \widehat{D}_{r+L}. \tag{9}$$

If L is deterministic then $E[\widehat{X}] = LE[\widehat{D}]$ [see Eq. (2)] and $Var[\widehat{X}] = LVar[\widehat{D}]$ [see Eq. (5)]. On the other hand when both daily demand and lead time are stochastic in nature then $\widehat{L}$ is a random variable in Eq. (9) and then $E[\widehat{X}] = E[\widehat{D}]E[\widehat{L}]$ [see Eq. (3)], $Var(\widehat{X}) = Var[\widehat{D}]E[\widehat{L}] + (E[\widehat{D}])^2 Var[\widehat{L}]$ [see Eq. (8)].

3. Notation and assumptions

3.1. Notations

In this model we consider n numbers of items. The following notations are used for i-th item to develop this model

h_i	holding cost per unit per unit time
p_i	per unit purchase cost
c_{2i}	per unit stockout cost
Q_i	order quantity (decision variable)
R_i	reorder point
$\widehat{D}_i$	average demand per unit time (random variable)
$\widehat{L}_i$	length of lead time (random variable)
$\widehat{A}_i$	Ordering cost per order (random variable)
$\widehat{X}_i$	demand during lead time (random variable)
B	maximum available budget for all items
x^+	maximum value of x and 0, i.e., x^+=max{x,0}
$E(.)$	mathematical expectation

3.2. Assumptions

The following are the assumptions used to formulate and solve an inventory problem.

(1) Shortage allowed and fully backlogged.
(2) Ordering cost $\widehat{A}_i$ is a random variable and is defined by $\widehat{A}_i = b_i + c_i\widehat{L}_i$, where b_i and c_i are real scalars. This function has been utilized in many researches (e.g., Chiu [15], Chen et al. [16] and Chang et al. [17]).
(3) The demand rate $\widehat{D}_i$, for i-th item, is a random variable, with mean $\mu_{\widehat{D}_i}$ and variance $\sigma^2_{\widehat{D}_i}$, having distribution function $F(d_i)$.
(4) The length of lead time $\widehat{L}_i$ is also randomly distributed with distribution function $G(l_i)$, having mean $\mu_{\widehat{L}_i}$ and variance $\sigma^2_{\widehat{L}_i}$.
(5) Demand during lead time (DDL), $\widehat{X}_i$ is convolution of demand rate and lead time (Ord [18], Mood et al. [19], Mayer [20], Kim et al. [21]). If demand rate $\widehat{D}_i$ and lead time $\widehat{L}_i$ be statistically independent, then mean and variance of demand during lead time are respectively (see Section 2)

$$E(\widehat{X}_i) = \mu_{\widehat{X}_i} = \mu_{\widehat{L}_i}\mu_{\widehat{D}_i} \quad \text{and} \quad var(\widehat{X}_i) = \sigma^2_{\widehat{X}_i} = \mu_{\widehat{L}_i}\sigma^2_{\widehat{D}_i} + \mu^2_{\widehat{D}_i}\sigma^2_{\widehat{L}_i}.$$

But if lead time is fixed, then

$$E(\widehat{X}_i) = \mu_{\widehat{X}_i} = L_i\mu_{\widehat{D}_i} \quad \text{and} \quad var(\widehat{X}_i) = \sigma^2_{\widehat{X}_i} = L_i\sigma^2_{\widehat{D}_i}.$$

(6) The reorder point R_i is the expected demand during lead time plus safety stock (SS), and SS $= k_i\times$ (standard deviation of lead time demand), i.e., $R_i = \mu_{\widehat{X}_i} + k_i\sigma_{\widehat{X}_i}$, where k_i is the safety factor and satisfying $P(\widehat{X}_i > R_i) = P(\widehat{Z}_i > k_i) = q_i, \widehat{Z}_i$ represents the standard normal random variable and q_i represents the allowable stock-out probability during lead time $\mu_{\widehat{L}_i}$.

4. Model formulation

By the above assumption the expected ordering cost of i-th item is $E(\widehat{A}_i)[= \mu_{\widehat{A}_i}]$. The expected net inventory level of i-th item at the end of a cycle is $R_i - \mu_{\widehat{X}_i}$ and at the beginning of the cycle is $Q_i + R_i - \mu_{\widehat{X}_i}$ and also the expected cycle length for i-th item is $\frac{Q_i}{E(\widehat{D}_i)}$. Therefore, the total random holding cost of i-th item for a single cycle is $h_i \int_0^{\frac{Q_i}{E(\widehat{D}_i)}} (Q_i + R_i - \mu_{\widehat{X}_i} - \widehat{D}_i t)dt$. So, the average expected holding cost for i-th item is $h_i(Q_i/2 + R_i - \mu_{\widehat{X}_i})$. Hence, the total expected cost per unit time of the system $TECU(Q_i, k_i)$ is as follows (see Fig. 1):

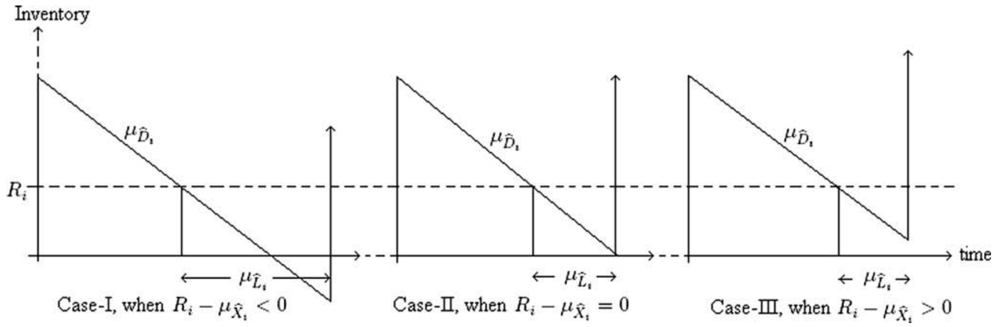

Fig. 1. The inventory situation when both demand rate and lead time are random.

$$\begin{aligned} TECU(Q_i, k_i) &= \textit{ordering cost} + \textit{holding cost} + \textit{stockout cost} \\ &= \sum_{i=1}^{n}\left[\frac{E(\widehat{D}_i)}{Q_i}E(\widehat{A}_i) + h_i\left(\frac{Q_i}{2} + R_i - \mu_{\widehat{X}_i}\right) + \frac{E(\widehat{D}_i)}{Q_i}c_{2i}E(\widehat{X}_i - R_i)^+\right] \\ &= \sum_{i=1}^{n}\left[h_i\left(\frac{Q_i}{2} + k_i\sigma_{\widehat{X}_i}\right) + \frac{\mu_{\widehat{D}_i}}{Q_i}\left\{\mu_{\widehat{A}_i} + c_{2i}E(\widehat{X}_i - R_i)^+\right\}\right] \end{aligned} \tag{10}$$

Here $E(\widehat{X}_i - R_i)^+$ is the expected stock-out quantity of i-th item which is given by

$$E(\widehat{X}_i - R_i)^+ = \int_{R_i}^{\infty}(x_i - R_i)h(x_i)dx_i \tag{11}$$

$$\text{or } E(\widehat{X}_i - R_i)^+ = \sum_{s}(x_{is} - R_i)h(x_{is}), \qquad \text{where } x_{ip} = R_i \quad \text{and} \quad s = p, p+1, p+2, \ldots \tag{12}$$

According to Tersine [2] the distribution of demand is normal at the factory level; the poisson, at retail level, and the negative exponential, at the wholesale and retail level. Also the distribution of lead time may be gamma, exponential, geometric and normal. Bagchi et al. [22] discussed elaborately on these topics. We relax the assumption about the distribution of demand rate lead time and demand during lead time. Only assume that

- the distribution function $F(d_i)$ of D_i belongs to the class of distribution function Γ_{d_i} with finite mean $\mu_{\widehat{D}_i}$ and standard deviation $\sigma_{\widehat{D}_i}$,
- the distribution function $G(l_i)$ of $\widehat{L}_i$ belongs to the class of distribution function Γ_{l_i} with finite mean $\mu_{\widehat{L}_i}$ and standard deviation $\sigma_{\widehat{L}_i}$,
- the distribution function $H(x_i)$ of $\widehat{X}_i$ belongs to the class of distribution function Γ_{x_i} with finite mean $\mu_{\widehat{X}_i}$ and standard deviation $\sigma_{\widehat{X}_i}$.

Lemma 1. *Gallego and Moon [14]*

$$E(\widehat{D} - Q)^+ \leqslant \frac{[\sigma^2 + (Q-\mu)^2]^{1/2} - (Q-\mu)}{2},$$

where Q is the overcapacity and $\widehat{D}$ is the random demand with mean μ and standard deviation σ.

Using the above lemma and for any $H(x_i) \in \Gamma_{x_i}$, it can be deduced that

$$E(\widehat{X}_i - R_i)^+ \leqslant \frac{1}{2}\left[\sqrt{\sigma^2_{\widehat{X}_i} + (R - \mu_{\widehat{X}_i})^2} - \left(R_i - \mu_{\widehat{X}_i}\right)\right] = \frac{1}{2}\left(\sqrt{1+k_i^2} - k_i\right)\sqrt{\mu_{L_i}\sigma^2_{\widehat{D}_i} + \mu^2_{\widehat{D}_i}\sigma^2_{\widehat{L}_i}}, \tag{13}$$

because $R_i = \mu_{\widehat{X}_i} + k_i\sigma_{\widehat{X}_i}$. Then Eq. (10) is reduced to

$$TECU(Q_i, k_i) \leqslant \sum_{i=1}^{n}\left[h_i\left(\frac{Q_i}{2} + k_i\sigma_{\widehat{X}_i}\right) + \frac{\mu_{\widehat{D}_i}}{Q_i}\left\{\mu_{\widehat{A}_i} + c_{2i}\frac{\sigma_{\widehat{X}_i}}{2}\left(\sqrt{1+k_i^2} - k_i\right)\right\}\right] \tag{14}$$

If the purchasing cost for i-th item is paid at the time of receiving the order, then the problem can be formulated by objective function with a random budget constraint, which is given below

$$\text{Minimize} \quad \left\{ \underset{H(x_i)\in\Gamma_{x_i}}{\text{Maximize}}\, TECU(Q_i,k_i) \right\}, \tag{15}$$

$$\text{Subject to} \quad \sum_{i=1}^{n} p_i(Q_i + R_i - \widehat{X}_i) \leqslant B, \tag{16}$$

where

$$\underset{H(x)\in\Gamma_x}{\text{Maximize}}\, TECU(Q_i,k_i) = \sum_{i=1}^{n}\left[h_i\left(\frac{Q_i}{2} + k_i\sigma_{\widehat{X}_i}\right) + \frac{\mu_{\widehat{D}_i}}{Q_i}\left\{ \mu_{\widehat{A}_i} + c_{2i}\frac{\sigma_{\widehat{X}_i}}{2}\left(\sqrt{1+k_i^2} - k_i\right)\right\}\right]. \tag{17}$$

There are several methods to solve the above problem. We consider a method known as Chance-Constrained programming technique, which is explained in Proposition 1.

Proposition 1 (Chance-constrained). *As the name indicates, the chance-constrained programming technique can be used to solve problems involving chance constraints, i.e., constraints having finite probability of being violated. This technique was originally developed by Charnes and Cooper [23]. If 'λ' is the probability of non-violation of the constraint (16) then the constraint can be written as*

$$P\left[\sum_{i=1}^{n} p_i(Q_i + R_i - \widehat{X}_i) \leqslant B\right] \geqslant \lambda. \tag{18}$$

Since, $\widehat{X}_i$ is the DDLT with finite mean $\mu_{\widehat{X}_i}$ and standard deviation $\sigma_{\widehat{X}_i}(>0)$ then the inequality (18) reduces to

$$\begin{aligned}
&\lambda \leqslant P\left(\sum_{i=1}^{n} p_i\widehat{X}_i + B \geqslant \sum_{i=1}^{n} p_i(Q_i+R_i)\right)\\
&\quad \leqslant \frac{E\left(\sum_{i=1}^{n} p_i\widehat{X}_i + B\right)}{\sum_{i=1}^{n} p_i(Q_i+R_i)} \text{(by Markov inequality)}\\
&\text{i.e.,} \sum_{i=1}^{n} p_iE(\widehat{X}_i) + B \geqslant \widetilde{\lambda}\sum_{i=1}^{n} p_i(Q_i+R_i)\\
&\quad \Rightarrow \sum_{i=1}^{n} p_i\mu_{\widehat{X}_i} + B \geqslant \lambda\sum_{i=1}^{n} p_i\left(Q_i + \mu_{\widehat{X}_i} + k_i\sigma_{\widehat{X}_i}\right)\\
&\quad \Rightarrow \sum_{i=1}^{n} p_i\mu_{\widehat{X}_i} + B \geqslant C\lambda, \quad \text{where } C = \sum_{i=1}^{n} p_i\left(Q_i + \mu_{\widehat{X}_i} + k_i\sigma_{\widehat{X}_i}\right).
\end{aligned} \tag{19}$$

Remark 1. If we take lead time as a constant but controllable then this model is converted to Ghalebsaz-Jeddi et al. [24] model.

Proof. Taking lead time as crisp variable, i.e., $\sigma_{\widehat{L}_i} = 0$ and $\mu_{\widehat{L}_i} = L_i$ then from assumption (5)

$$E(\widehat{X}_i) = \mu_{\widehat{X}_i} = L_i\mu_{\widehat{D}_i} \quad \text{and} \quad var(\widehat{X}_i) = \sigma^2_{\widehat{X}_i} = L_i\sigma^2_{\widehat{D}_i}. \qquad \square$$

This result incurs that the optimization problem (15) with constraint (18) is similar to Ghalebsaz-Jeddi et al. [24] model.

But it will be more adequate if λ be a fuzzy number, in particular let it be a triangular fuzzy number and is defined by $\widetilde{\lambda} = (\lambda - \epsilon, \lambda, \lambda + \epsilon)$, where ϵ is the positive real satisfying $\lambda + \epsilon \leqslant 1$ and $\lambda \in [0,1]$, then the above problem is stated as follows:

$$\text{Minimize} \quad \left\{ \underset{H(x_i)\in\Gamma_{x_i}}{\text{Maximize}}\, TECU(Q_i,k_i) \right\}, \tag{20}$$

$$\begin{aligned}
\text{Subject to} \quad & P\left(\sum_{i=1}^{n} p_i(Q_i + R_i - \widehat{X}_i) \leqslant B\right) \geqslant \widetilde{\lambda},\\
& \Rightarrow \sum_{i=1}^{n} p_i\mu_{\widehat{X}_i} + B \geqslant C\widetilde{\lambda}
\end{aligned} \tag{21}$$

Proposition 2 (Surprise). *Denoting $\sum_{i=1}^{n} p_i \mu_{\widehat{X_i}} + B$ by ξ then the inequality (21) is of the form $\xi \geqslant C\widetilde{\lambda}$. By using the possibility distribution the fuzzy membership function is as follows:*

$$Pos\left(\xi \geqslant C\widetilde{\lambda}\right) = \begin{cases} 0, & \text{for } \xi \leqslant C(\lambda - \epsilon), \\ \frac{\xi - C(\lambda-\epsilon)}{\epsilon C}, & \text{for } C(\lambda - \epsilon) < \xi < C\lambda, \\ 1, & \text{for } \xi \geqslant C\lambda. \end{cases} \tag{22}$$

Next we obtained the surprise function s_ξ from the membership function $Pos(\xi \geqslant C\widetilde{\lambda})$ using the following relation:

$$s_\xi = \left[Pos\left(\xi \geqslant C\widetilde{\lambda}\right)^{-1} - 1 \right]^2. \tag{23}$$

Hence the above minimization problem (20) subject to the constrained (21) is reduced to a multi-objective problem as follows:

$$Minimize \left\{ \underset{H(x)\in\Gamma_x}{Maximize}\, TECU(Q_i, k_i), s_\xi(Q_i, k_i) \right\}, \tag{24}$$

$$where\ s_\xi(Q_i, k_i) = \left[\frac{\lambda_2 \sum_{i=1}^{n} p_i (Q_i + \mu_{\widehat{X_i}} + k_i \sigma_{\widehat{X_i}})}{\sum_{i=1}^{n} p_i \mu_{\widehat{X_i}} + B - \lambda_1 \sum_{i=1}^{n} p_i (Q_i + \mu_{\widehat{X_i}} + k_i \sigma_{\widehat{X_i}})} \right]^2.$$

Remark 2. When $s_\xi(Q_i, k_i) \to 0$ then the multi-objective function (24) is converted to the objective function (15) with constraint (18).

Proof. $s_\xi(Q_i, k_i) \to 0 \Rightarrow Pos(\xi \geqslant C\widetilde{\lambda}) \to 1 \Rightarrow \xi \geqslant C\lambda$, i.e., $\sum_{i=1}^{n} p_i \mu_{\widehat{X_i}} + B \geqslant C\lambda \Rightarrow$ the multi-objective objective function (24) is equivalent to the objective function (15) with constraint (19), i.e., the constraint (18). □

5. Solution algorithm

To solve the above multi-objective problem, let the number of types of products controlled by the inventory system is two, i.e., $n = 2$.

Step-1: Finding the values of Q_1, Q_2, k_1 and k_2 such that the value of $s_\xi(Q_1, Q_2, k_1, k_2)$ is minimum. But it does not give the minimum value of $Maximize_{H(x)\in\Gamma_x} TECU(Q_1, Q_2, k_1, k_2)$. So we have to find a compromise Q_1^*, Q_2^*, k_1^* and k_2^* which give the next minimum of $s_\xi(Q_1, Q_2, k_1, k_2)$ and $Maximize_{H(x)\in\Gamma_x} TECU(Q_1, Q_2, k_1, k_2)$, i.e.,

$$Minimum\{s_\xi(Q_1, Q_2, k_1, k_2)\} \leqslant s_\xi(Q_1^*, Q_2^*, k_1^*, k_2^*)$$
$$Minimum\{Maximize_{H(x)\in\Gamma_x} TECU(Q_1, Q_2, k_1, k_2)\} \leqslant Maximize_{H(x)\in\Gamma_x} TECU(Q_1^*, Q_2^*, k_1^*, k_2^*).$$

Step-2: The Q_1^*, Q_2^*, k_1^* and k_2^* are obtained by minimizing the function $Z(Q_1, Q_2, k_1, k_2)$ which is defined by

$$Z(Q_1, Q_2, k_1, k_2) = Maximize_{H(x)\in\Gamma_x} TECU(Q_1, Q_2, k_1, k_2) + s_\xi(Q_1, Q_2, k_1, k_2),$$
i.e. $Minimum Z(Q_1, Q_2, k_1, k_2) = Z(Q_1^*, Q_2^*, k_1^*, k_2^*)$.

Step-3: The solution $(Q_1^*, Q_2^*, k_1^*, k_2^*)$ is known as the pareto-optimal solution of the multi-objective problem (24).

6. Numerical example

To illustrate this model by an example we consider two different types of products, for which the following data have been used. $h_1 = \$3$ per unit per month, $h_2 = \$3.5$ per unit per month, $c_{21} = \$10$ per unit, $c_{22} = \$8.5$ per unit, $p_1 = \$55$ per unit,

Table 1
Distributions of monthly demand and lead time in days.

Monthly-demand d_{1i}	Probability $f(d_{1i})$	Lead time l_{1j}	Probability $g(l_{1j})$	Monthly-demand d_{2i}	Probability $f(d_{2i})$	Lead time l_{2j}	Probability $g(l_{2j})$
385	0.06	4	0.10	530	0.09	1	0.10
393	0.15	5	0.23	540	0.15	2	0.15
398	0.31	6	0.35	549	0.26	3	0.50
405	0.35	7	0.30	554	0.30	4	0.25
412	0.13	8	0.02	557	0.20		

Table 2
Optimal results for two products inventory controlled problem.

$Z(Q_1, Q_2, k_1, k_2)$	Upper bound of *TECU*	Surprise	Q_1	Q_2	k_1	k_2
1240.792	1240.324	0.4684276	146.46	177.02	1.25	1.08

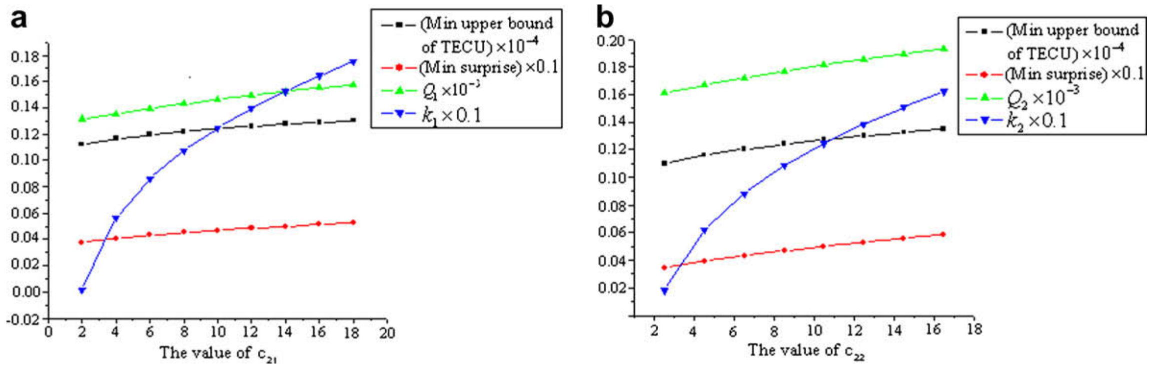

Fig. 2. (a) Graphical representation of the Min upper bound of TECU, Min Surprise, Q_1 and k_1 for different values of c_{21}. (b) Graphical representation of the Min upper bound of TECU, Min Surprise, Q_2 and k_2 for different values of c_{22}.

$p_2 = \$77$ per unit and $\widetilde{\lambda} = (0.85, 0.9, 0.95)$ and maximum available budget is $B = \$65000$. The two independent distributions of monthly demand and lead time in days are shown in Table 1.

Using this distribution the mean demand and mean lead time are respectively $\mu_{D_1} = \sum_{i=1}^{5} d_{1i} f(d_{1i}) = 400$ units/month and $\mu_{L_1} = \sum_{j=1}^{4} l_{1j} g(l_{1j}) = 6$ days, $\mu_{D_2} = \sum_{i=1}^{5} d_{2i} f(d_{2i}) = 550$ units/month and $\mu_{L_2} = \sum_{j=1}^{3} l_{2j} g(l_{2j}) = 3$ days, consequently the standard deviations of demand rate and lead time are $\sigma_{D_1} = \left[\sum_{i=1}^{5} (d_{1i} - \mu_{D_1})^2 f(d_{1i})\right]^{1/2} = 7$ units/month and $\sigma_{L_1} = \left[\sum_{j=1}^{4} (l_{1j} - \mu_{L_1})^2 g(l_{1j})\right]^{1/2} = 1$ day $\sigma_{D_2} = \left[\sum_{i=1}^{5} (d_{2i} - \mu_{D_2})^2 f(d_{2i})\right]^{1/2} = 8$ units/month and $\sigma_{L_2} = \left[\sum_{j=1}^{3} (l_{2j} - \mu_{L_2})^2 g(l_{2j})\right]^{1/2} = 1$ day. Then the expected setup costs are $\mu_{A_1} = \$45$ per setup, $\mu_{A_2} = \$54$ per setup and the maximum available budget for this model is $\$65,000$. Using the solution technique given in Section 4, the optimal results for two products inventory controlled problem are as follows (see Table 2).

The unit shortage cost has an important effect on the cost function. At first increasing the unit shortage cost for the first item (i.e., c_{21}) when the unit shortage cost for second item (i.e., c_{22}) is fixed, the value of safety factor (i.e., k_1) increases; it is obvious because the management tries to reduce shortages (i.e., $E(X - R)^+$) by increasing shortage cost. Consequently the order quantity (i.e., Q_1) is increased. Since Surprise (i.e., s_ξ) increases with increase in unit shortage cost, the total expected cost per unit time (i.e., TECU) must increase. The same result is obtained if we increase the unit shortage cost for second item. Both the results are shown in Fig. 2a and b, respectively.

7. Concluding remarks

The present paper proposes a solution procedure for a multi-item mixture inventory model in which safety factor and order quantity are considered as the decision variables. After addition of budgetary constraint the optimization problem is transformed into a multi-objective optimization problem with the help of fuzzy chance-constrained programming technique and surprise function. Some of the advantages of the surprise function approach are as follows: (i) it has a consistent semantic interpretation. (ii) It allows the joint use of quantitative and qualitative knowledge, using simple rules of logic. (iii) It is a direct extension of (and allows combination with) the least square approach to reconciling conflicting approximate numerical data. (iv) It is ideally suited to optimization under imprecise or conflicting goals, specified by a combination of soft and hard interval constraints. (v) It gives a straightforward approach to constructing families of function consistent with fuzzy associative memories as used in fuzzy control, with tuning parameters (reflecting linguistic ambiguity) that can be adapted to available performance data. Here, minimax distribution free procedure is applied to solve the present problem.

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